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ESTIMATE OF FRAGMENT-FORMATION IN THE DESTRUCTION OF A SPHERICAL SHELL

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The study of the unsteady motion of metallic shells up to rupture under the effect of intensive loads is of practical interest. Papers of theoretical and experimental nature are known [1-5] in which the question of expansion of a spherical shell under the effect of explosive loads has been examined.

In this paper the law of the unsteady motion of a hollow sphere subjected to variable internal pressure or an initial velocity field is determined in the scheme of an isotropic incompressible viscoplastic medium. In the case of ideal plasticity the shell rupture time is determined. A formula is derived to estimate the quantity of fragments. The results obtained are compared with known experimental data.

1. Formulation of the Problem

A hollow sphere subjected to a variable internal pressure or an initial velocity field expands nonstationarily for given initial data. On the outer boundary of the sphere there is no motion. The shell material is isotropic, incompressible, and satisfies the relations of a viscoplastic medium.

For central symmetry of the sphere deformation, we have the following equations for the stress tensor components σ_r, σ_θ, σ_φ, the radial component of the velocity vector v in the spherical coordinates r, θ, φ outside the mass force field:

Equation of motion of a continuous medium

$$\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}(2\sigma_r-\sigma_\theta-\sigma_\varphi)=\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}\right); \tag{1.1}$$

Continuity equation

$$\frac{\partial}{\partial r}(r^2v)=0; \tag{1.2}$$

The relationships of a viscoelastic medium [6] in spherical coordinates with central symmetry

$$\begin{gathered}\sigma_r=\sigma-\frac{2}{3}\sigma_s+\mu\frac{\partial v}{\partial r},\quad \sigma=\frac{1}{3}(\sigma_r+\sigma_\theta+\sigma_\varphi),\\ \sigma_\theta=\sigma_\varphi=\sigma+\frac{1}{3}\sigma_s+\mu\frac{v}{r}.\end{gathered} \tag{1.3}$$

Here ρ is the density of the sphere material; σ_s, dynamic yield point; μ, dynamic coefficient of viscosity; and $t \geq 0$, time.

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The following conditions are satisfied on the boundaries of the spherical shell $r = R_i$ ($i = 1, 2$, $R_1 < R_2$):

Dynamic

$$\sigma_r = -p(t) \quad \text{at} \quad r = R_1, \ \sigma_r = 0 \quad \text{at} \quad r = R_2; \tag{1.4}$$

Kinematic

$$dR_i/dt = v \quad \text{at} \quad r = R_i. \tag{1.5}$$

The nonstationary expansion of a sphere under the effect of an explosive load is considered. Two kinds of motion are later distinguished. The first, when the sphere motion starts from a state of rest under the effect of variable internal pressure $p(t) = p_0(R_{10}/R_1)^{3\gamma}$, where $\gamma > 1$ is the isentropic index of the detonation products. In this case the initial conditions for $t = 0$ have the form

$$R_i = R_{i0}, \ dR_i/dt = 0, \ p = p_0 \neq 0. \tag{1.6}$$

The second motion is inertial, when the initial velocity field is given. There is no pressure within the sphere. We then have for $t = 0$

$$R_i = R_{i0}, \ p = 0, \ dR_i/dt = V_{i0} \neq 0. \tag{1.7}$$

The equalities (1.1)-(1.6) or (1.7) uniquely determine the problem of the nonstationary motion of a spherical shell from an incompressible viscoelastic material.

2. Law of Variation of the Sphere Boundaries

Substituting (1.3) into the motion equation (1.1) and taking account of the value of the first integral in (1.2), we obtain a first degree differential equation in the mean stress, which is then integrated with respect to the variable r. Having determined the value for σ, v in this manner from (1.3) and (1.4), we obtain the solution for the stress tensor components σ_r, σ_θ, σ_φ, and the law of variation of the sphere boundaries is hence determined because of (1.4)-(1.6).

Let us introduce dimensionless variables and parameters for the initial data of (1.6) by means of the formulas

$$\begin{gathered}\overline{R}_i = R_i/R_{10}, \quad \overline{t} = t\sqrt{p_0/\rho}/R_{10}, \quad \sigma_* = \sigma_s/p_0, \quad \nu = \mu/R_{10}\sqrt{p_0\rho}, \\ \varkappa = R_2/R_1, \quad \varkappa_0 = R_{20}/R_{10}.\end{gathered} \tag{2.1}$$

In the case of inertial motion of the sphere with the initial conditions (1.7), the substitution $p_0 \to \rho V_{10}^2$ must be made in (2.1). The bar is henceforth omitted over the dimensionless quantities.

Then to determine the change in the inner radius of the sphere subjected to variable internal pressure, we obtain a Cauchy problem in the form

$$\ddot{R}_1 + a_1\dot{R}_1^2 + a_2\dot{R}_1 + a_3 = 0, \ R_1(0) = 1, \ \dot{R}_1(0) = 0, \tag{2.2}$$

where

$$\begin{gathered}a_1 = R_1^{-1}\left[2 - \frac{(1-\varkappa^{-4})}{2(1-\varkappa^{-1})}R_1^{-2}\right]; \ \varkappa = \left[1 + \frac{(\varkappa_0^3 - 1)}{R_1^3}\right]^{1/3}; \\ a_2 = \frac{2\nu}{R_1^2}\frac{(1-\varkappa^{-3})}{(1-\varkappa^{-1})}; \ a_3 = \frac{\varkappa R_1^{-1}}{\varkappa - 1}(2\sigma_*\ln\varkappa - R_1^{-3\gamma});\end{gathered}$$

and the point denotes differentiation with respect to time. For $\nu = 0$ the problem reduces to one that is known [1].

Let us consider a thin-walled sphere when $\varkappa_0 = 1 + \varepsilon_0$, $\varepsilon_0 = \delta_0/R_{10} \ll 1$, δ_0 is the initial thickness of the shell wall. In this case from (2.2) we obtain a second order differential equation containing a small parameter in the highest derivative. The method of constructing the solution of the singular problem in a small parameter is known (see [7], for instance). We make the substitution

$$\dot{R}_1 = V_1, \ \ddot{R}_1 = V_1 dV_1/dR_1. \tag{2.3}$$

Then (2.2) reduces to an Abel equation of the second kind. To first order accuracy in ε_0 for a thin-walled shell we obtain the Cauchy problem in the form

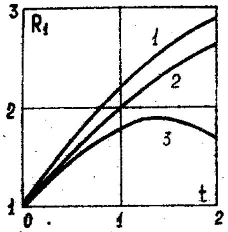

Fig. 1

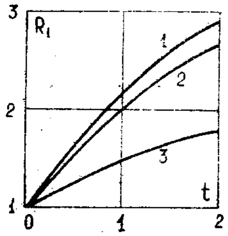

Fig. 2

$$\varepsilon_0 V_1 \frac{dV_1}{dR_1} + 2\varepsilon_0 R_1^{-1} V_1 (2V_1 - 3\nu R_1^{-1}) + R_1^{-1}(R_1^{3(1-\gamma)} - 3\varepsilon_0 \sigma_*) = 0, \quad V_1 = 0 \text{ at } R_1 = 1 \tag{2.4}$$

because of (2.3).

In the case when the sphere material corresponds to the ideal plasticity condition ($\nu = 0$), from (2.4) we have the solution [8] in the form

$$V_1 = (1 - R_1^{-4}) \left[2 \int_1^{R_1} (x^{-4} - 1)^{-2} (3\sigma_* - \varepsilon_0^{-1} x^{3(1-\gamma)}) x^{-1} dx \right]^{1/2}. \tag{2.5}$$

The integral in (2.5) is expressed in elementary function for specific $\gamma > 1$, integers, for instance. We hence obtain a law of variation of the sphere inner boundaries in quadratures. Construction of the solution for the more general case [Cauchy problem (2.2)] was by the Runge–Kutta numerical method. As an illustration, the numerical computation of the problem (2.2) is represented in Fig. 1 for the dimensionless quantities R_1, t for the fixed parameters $\nu = 0.1$, $\varkappa_0 = 1.20$, $\gamma = 2.5$. Curves 1-3 in Fig. 1 correspond to the values of the dimensionless parameter $\sigma_* = 0, 0.1, 1.0$.

The nature of the change in the inner radius of the spherical shell with respect to time is shown in Fig. 2 for different values of the parameter ν: 1) $\nu = 0$, 2) 0.1, 3) 1.0. The remaining parameters are fixed $\sigma_* = 0.1$, $\varkappa_0 = 1.20$, $\gamma = 2.5$. From the computations there also follows that the rate of sphere expansion diminishes when the parameters $\varkappa_0$, γ increase.

Now, let the sphere expand by inertia with the initial data (1.7). In this case the law of variation of the sphere radius R_1 is determined from the analogous Cauchy problem (2.2), where the variable coefficient a_3 takes the form $a_3 = 2\sigma_* \varkappa \ln \varkappa / [(\varkappa - 1)R_1]$. For a thin-walled shell, because of (2.3) we obtain to first-order accuracy in ε_0 a Cauchy problem to determine R_1 for inertial expansion of the sphere in the form

$$V_1 \frac{dV_1}{dR_1} + 2R_1^{-1} V_1 (2V_1 - 3\nu R_1^{-1}) - 3\sigma_* R_1^{-1} = 0, \; V_1 = 1 \text{ for } R_1 = 1. \tag{2.6}$$

Here in expanding the coefficients a_j (j = 1, 2, 3) in a series in the small parameter ε_0, this latter enters the differential equation (2.6) in a regular manner [7] starting with the power ε_0^2. The solution of problem (2.6) for $\nu = 0$ has the form (see [8], for example)

$$V_1 = 1 - \sqrt{\tfrac{3}{2}\sigma_*}\,(1 - R_1^{-4}) \{[(R_1^4 - 1)^{-1} - \ln (R_1^4 - 1)]^{1/2}. \tag{2.7}$$

The law of motion of the inner radius of the sphere is determined in quadratures from (2.5) and (2.7) by the formula

$$t = \int_1^{R_1} \frac{dx}{V_1(x)}. \tag{2.8}$$

Let us note that (2.5) and (2.7) express the energy conservation law. For instance, after raising both sides to a square, from (2.5) we obtain the following energetic relationship per unit mass

$$I_0 + \frac{V_1^2}{2} = I_1,$$

where $I_0 = (1 - R_1^{-4})^2 \varepsilon_0^{-1} \int_1^{R_1} (1 - x^{-4})^{-2} x^{2-3\gamma} dx$ is the work of explosion products; $V_1^2/2$, kinetic component of the sphere energy; $I_1 = (1 - R_1^{-4})^2 \quad 3\sigma_* \int_1^{R_1} (1 - x^{-4})^{-2} x^{-1} dx$, work of plastic deformation.

In particular, formulas to determine the inner radius of the sphere at the time of its maximal expansion ($V_1 = 0$) can be obtained from (2.5) and (2.7) without disturbing the continuity. Approximately they have the form

For a sphere under pressure (for $\gamma = 3$)

$$R_{1p} = R_{10}\left[1 + \frac{1}{2}\left(1 - \frac{p_0 R_{10}}{3\sigma_s \delta_0}\right)^{-1}\right];$$

For inertial motion of the sphere

$$R_{1u} = R_{10}\left(1 + \frac{\rho V_{10}^2}{6\sigma_s}\right).$$

Certain destruction of the sphere ($R_{1p} \to \infty$) follows from (2.8) in the case of compliance with the relationship

$$p_0 \geqslant 3\sigma_s \delta_0 / R_{10}.$$

The connection between the values of R_1 and R_2 of a thin-walled sphere is expressed by the relationship

$$R_2 \simeq R_1\left(1 + \frac{\varepsilon_0}{R_1^3}\right). \tag{2.9}$$

To the same degree of approximation we obtain from (1.2) and (1.5) for the velocities $V_1 = \dot{R}_1$, $V_2 = \dot{R}_2$.

$$V_2 \simeq V_1 (1 - 2\varepsilon_0 / R_1^3). \tag{2.10}$$

3. Destruction of the Spheres

The time criterion for preparing a body for destruction and the integrated time criterion for total joining of the cracks on the basis of the equation of nonstationary crack growth should be satisfied [9] for total destruction of a solid over the section under consideration. The dynamic criterion for crack joining and total destruction reduces to the integral relationship [10]

$$\int_0^\tau qc\,dt = \frac{\alpha}{1-k} \ln \frac{1}{S_0}, \tag{3.1}$$

where τ is the time of destruction; q, density of the energy being liberated by the sound wave; c, sound speed; α, work to form unit crack area; k, reflection coefficient of the acoustic wave enegy flux averaged over time; S_0, initial fraction of the area of the body section under consideration that is overlapped by cracks. The effective energy of dynamic destruction $\alpha_* = (\alpha \ln S_0)/(k - 1)$ is determined from experiment.

Taking account of the volume of the problem of sphere expansion for $q = \sigma_\theta^2/E$, by analogy with a ring [11] we obtain the following integral equation from (3.1)

$$\int_0^{\tau_*} \sigma_\theta^2 R_2^2 (1 - R_1^2/R_2^2)\, dt = \beta R_{2*}^2 (1 - R_{1*}^2/R_{2*}^2),\ \beta = \frac{\alpha_* E \sqrt{p_0/\rho}}{c p_0^2 R_{10}}, \tag{3.2}$$

where E is Young's modulus; R_{1*}, R_{2*}, values of the sphere radii at the time of destruction; $\tau_* = \tau \sqrt{p_0/\rho} \cdot (R_{10})^{-1}$, dimensionless time of shell destruction under the effect of internal pressure; and $\tau_* = \tau V_{10}/R_{10}$, inertial case. The circumferential stress σ_θ of a viscoplastic sphere on the outer boundary has the following form because of Sec. 1 and (2.1)

$$\sigma_\theta = \sigma_* + 3\nu \dot{R}_2 / R_2. \tag{3.3}$$

Let us consider a thin-walled sphere when $\varkappa_0 = 1 + \varepsilon_0$, $\varepsilon_0 \ll 1$. To first-order accuracy, we have

$$1 - R_1^2/R_2^2 \simeq 2\varepsilon_0 / R_2^3.$$

Taking account of (3.2) and (3.3), we hence obtain

$$\int_0^{\tau_*} (\sigma_*^2 + 6\nu\sigma_*\dot{e} + 9\nu^2\dot{e}^2)\, R_2^{-1} dt = \beta R_{2*}^{-1},\ \dot{e} = \dot{R}_2/R_2. \tag{3.4}$$

Let us examine (3.4) under the assumption that the true strain rate is a constant ($\dot{e}$ = const). Then we arrive at the following relationship for the plastic deformation $e = R_{2*}/R_{20} - 1$ from (3.4)

$$e = \beta\dot{e}\,(\sigma_* + 3\nu\dot{e})^2. \tag{3.5}$$

Analyzing (3.5), we note the existence of a maximum for the plastic deformation equal to $e_{max} = \beta/12\nu\sigma_*$, for $\dot{e} = \sigma_*/3\nu$. This singularity of the dynamic behavior of viscoplastic media (the dynamic peak of plasticity) was first obtained and interpreted by test data in the case of explosive deformation of tubes in [11]. For axisymmetric viscoplastic deformations of a tube under the effect of an internal load, the circumferential stress on the outer boundary is expressed by the formula $\sigma_\theta = \sigma_* + 4\nu\dot{R}_2/R_2$ in our notation [12]. Then, repeating the preceding computations, from (3.2) we obtain for the plastic deformation of a tube analogous to (3.5)

$$e = \beta\dot{e}\,(\sigma_* + 4\nu\dot{e})^2. \tag{3.6}$$

The relationships (3.5) and (3.6) confirm the deduction formulated in [11] that the existence of a dynamic plasticity peak is general in nature for the destruction of a shell and the simplest structures fabricated from viscoplastic materials.

4. Estimate of Fragment Formation

At the present time a quantitative description of the rate and time of destruction of a sphere by using the energetic criterion (3.2) is problematical because of the absence of test data for α_*. Here we use the simplest criterion for destruction in the achievement of the limiting plastic deformation $e_* = R_{1*}/R_{10} - 1$ with the energetic relationships (2.5) and (2.7) taken into account. Such an approach yielded good quantitative estimates for the description of explosive destruction of metal rings (see [13, 14], for instance).

Because of (2.1), we have $R_{1*} = 1 + e_*$ on the inner boundary of the sphere at the time of destruction. Taking the approximation $\ln[(1 + e_*)^4 - 1] \simeq \ln 4e_*$, $e_* \ln 4e_* \simeq -e_*$, $[1 - (1 + e_*)^{-4}] \simeq 4e_*$, for $\gamma = 3$, we obtain from (2.5) and (2.8) for the value of the velocity of the inner boundary at the time of destruction and for the time of destruction, respectively

$$V_{1*} = \sqrt{\frac{2e_*}{\varepsilon_0}}\,[1 + 3\sigma_*\varepsilon_0 - 4(1 - 3\sigma_*\varepsilon_0)\, e_*]^{1/2},\ \tau_1 = \sqrt{2\varepsilon_0 e_*}\,(1 + \sigma_*\varepsilon_0)^{1/2}. \tag{4.1}$$

In the case of inertial expansion of the sphere in the ideal plasticity scheme, we have from (2.7) and (2.8) (to the same accuracy)

$$V_{1*} = 1 - \sqrt{6\sigma_* e_*},\ \tau_1 = e_*\left[1 + 2\sqrt{\tfrac{2}{3}\sigma_* e_*}\right]. \tag{4.2}$$

The time when the destruction front reaches the outer shell boundary is determined by the equality

$$\tau_2 = \int_{\varkappa_0}^{R_{2*}} \frac{dx}{V_2(x)},$$

Because of (2.9) and (2.10) we hence obtain to first order accuracy in ε_0, that $\tau_1 = \tau_2 = \tau_*$. Here we note the relationship $e = e_*(1 - 3\varepsilon_0)$, $e = R_{2*}/R_{20} - 1$.

For later we make specific the nature of sphere destruction by analogy with an annular shell [13]. Namely, we assume that destruction occurs by using radial crack formation, and the equality $l = c\tau$ holds for the estimate of the characteristic dimension of the fragments, where in conformity with the dimensionality $\tau = \tau_* R_{10}/(p_0/\rho)^{1/2}$ in the case of destruction of a sphere under the effect of internal pressure, and $\tau = \tau_* R_{10}/V_{10}$ in the inertial case.

In substance, the value of l characterizes the diameter of the base of spherical segments into which the shell is ruptured. The quantity of fragments n is estimated by the ratio between the sphere surface $f = 4\pi R_{2*}^2$ and the side surface of the spherical segment $f_* = 2\pi R_{2*}[R_{2*} - (R_{2*}^2 - l^2/4)^{1/2}]$; hence,

$$n = f/f_* = \left[1 - \sqrt{1 - l^2/4R_{2*}^2}\right]^{-1} \simeq 8R_{2*}^2/l^2. \tag{4.3}$$

TABLE 1

Sphere material	e	c, m/sec	$\rho\cdot 10^3$, kg/m^3	n_p	n_u
Duraluminum σ_s=0,29 GPa	0,52	5110	2,71	94	111
Copper σ_s = 0.22 GPa	0,57	3980	8,89	46	165
Titanium σ_s =0.37 GPa	0,35	4847	4,51	74	219
Zinc σ_s = 0.12 GPa	0,52	3700	7,14	68	223

TABLE 2

Sphere material	e	$\rho\cdot 10^3$, kg/m^3	τ, μsec	τ_e, μsec	τ_e/τ
Duraluminum σ_s = 0.29 GPa	0,52	2,71	9,91	10,0	1,01
Copper σ_s = 0.22 GPa	0,57	8,89	19,44	19,5	1,00
Steel σ_s = 0.32 GPa	0,33	7,85	14,12	16,0	1,13

TABLE 3

$R_{10}\cdot 10^{-3}$, m	$\delta_0\cdot 10^{-3}$, m	V_{10}, m/sec	V_{20}, m/sec	e,%	τ, μsec	τ_e, μsec
153	2,6	43,4	42	3,5	303	400
153	2,6	30,0	29	1,8	231	200
38,1	3,4	87,8	74	4,7	37	30
38,1	10,4	132,8	82	4,4	20	34

From (2.1) and (2.9) there follows

$$R_{2*} = R_{10}(1+e_*)[1+\varepsilon_0/(1+e_*)^3];$$

then because of (4.1)-(4.3) we obtain relationships to estimate the quantity of fragments during destruction of the sphere under the effect of internal pressure n_p of an initial velocity field n_u, respectively

$$n_p = \frac{4p_0(1+e_*)^2}{\rho c^2\varepsilon_0 e_*}\left[1+\left(\frac{2}{(1+e_*)^3}-\sigma_*\right)\varepsilon_0\right],\ \sigma_* = \sigma_s/p_0; \tag{4.4}$$

$$n_u = 2\left[\frac{2(1+e_*)V_{10}}{\varepsilon_0 e_* c}\left(\frac{1+\varepsilon_0/(1+e_*)^3}{1+2\sqrt{\frac{2}{3}\sigma_* e_*}}\right)\right]^2,\ \sigma_* = \sigma_s/\rho V_{10}^2. \tag{4.5}$$

In particular, an increase in the quantity of fragments as the initial parameters R_{10}, p_0, V_{10} increase follows from (4.4) and (4.5), which agrees with the general deductions in [5, 13]. As an illustration, the computation of the quantity of fragments by means of (4.4) and (4.5) is represented in Table 1. Here the limiting quantity e and the value of σ_s are written in conformity with the experimental investigations [1] for the destruction of a spherical shell of radius R_{10} = 0.115 m with a wall thickness of δ_0 = 0.005 m (ε_0 = 0.043). A sphere of different material subjected to an explosive substance (hexogene) with an initial pressure of p_0 = 15.7 GPa is expanded intensively to destruction. In the case under consideration, the explosive fills the inner volume of the shell completely. The computation of the quantity of fragments by means of (4.5) is realized for V_{10} = 300 m/sec.

There are no experimental and theoretical data in [1] on an estimate of the quantity of fragments during destruction of the shells being investigated, but experimental values of the time to destroy spheres of duralumin, copper, and low-alloyed steel are given, respectively: $\tau_e = 10.0$, 19.5, 16.0 μsec. Let us compare these results with the computed values obtained by means of (4.1) in dimensional form with (2.1) taken into account. Initial data on σ_s, ρ are represented in Table 2 for the shell material, as are also experimental values for the limiting plastic deformation e, the time of destruction τ_e according to [1], and the computed τ. Here the geometric dimensions of the shell are $R_{10} = 0.115$ m, $\delta_0 = 0.005$ m for an initial pressure of $p_0 = 15.7$ GPa.

We now turn to experiments [4] when the maximum velocity of destruction of the outer shell surface, which corresponds to V_{20} in our case, the time of destruction τ_e, and the limiting magnitude of the deformation e, were recorded for explosive loading of spherical shells. The explosive charge (50/50 TG) of density $\rho_* = 1.65 \cdot 10^3$ kg/m^3 in the form of a sphere was placed in and initiated at the center of the shell. In all the tests there was a substantial gap between the charge and the shell inner surface, which corresponds to pulse loading. The motion of the sphere boundaries is here realized by inertia. Experimental τ_e and computed τ results of the destruction time are represented in Table 3 together with the initial data. The determination of τ was by means of (4.2) with (2.1) taken into account. The vessels considered here were fabricated from steel 35 with different initial geometric parameters. The mechanical properties of the material were determined in [4] for quasistatic extension of the specimens: $\sigma_0 = 0.29$ GPa is the yield point, and $\sigma_b = 0.54$ GPa is the strength. Hence, according to the results of [15], the dynamic yield point is $\sigma_s = 0.345$ GPa. For a known initial loading rate of the outer shell surface V_{20} the velocity V_{10} needed for the computation was determined from the relationship in the initial parameter $V_{10} = \varkappa_0^2 V_{20}$, known for a sphere.

As follows from Table 1, the actual appraisable results on determining the quantity of fragments are obtained for explosive loading of spherical shells. In particular, the quantitative relations in the time of destruction of a sphere when it expands intensively until rupture under the effect of variable internal pressure or an initial velocity field, are in satisfactory agreement with experiment.

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